Piecewise Cumulative Weibull Modelling of Radar Cross Section

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Abstract

Statistical models of radar cross section in radar system simulation are common for providing data to test the interaction of the target tracking systems and the decision making activities. For many decades, the scintillation of the target RCS has been represented using models developed by Swerling. This paper investigates how well the Swerling models relate to the RCS of modern targets, and outlines a computationally simple approach to generating representative random RCS values based on a piecewise approximation of the target RCS cumulative distribution function that employs two sections of a Weibull distribution, and also provides simple mechanisms to mimic the temporal correlation effects of scintillation seen with target manoeuvre and the variation of RCS with range.

1 Introduction

The fidelity requirements of radar models is very dependent on the particular radar simulation application. For simulation and testing of surveillance radar systems and target trackers, representative detection data must be generated in order to allow the impact of missed detection to be assessed fully. For speed, statistical models are often employed where the fluctuation of the radar cross section is used to determine if the radar is likely to detect the target at the current range. Many statistical target models exist, but often detection behaviours based on the Swirling models [1] are employed as they provide results that have a known theoretical basis and are widely accepted in industry. The Swirling models were developed from observations and radar measurements of 'traditional' targets and also with radar hardware that may use quite different processing to modern systems.

For detection of aircraft at long ranges, the vehicles are often observed from an aspect that approximates well to just the horizontal plane. If the vehicle manoeuvres however (e.g. a roll manoeuvre that creates a bank during a hard turn), then it is possible that the vehicle could be observed from any aspect over a full 4π Steradians. Additionally, although many aircraft are still of 'conventional' design, there is an increasing trend towards platforms that have a reduced radar cross section.

This paper uses electromagnetic simulation of representative vehicle structures over a full 4π Steradians and investigates how the statistics of the fluctuation of the radar cross section of conventional and low observable targets varies with view angle, and in particular, analyses the fluctuation in comparison to the traditional Swirling model statistics. As the statistical distributions may sometimes be very different from the traditional Swerling models, a simple alternative approach is presented based using a piecewise approximation with two different Weibull distribution sections, which allows representative statistics to be mimicked. The use of a piecewise Weibull approximation leads to a simple method of creating temporally correlated data to mimic the RCS scintillation effects observed with target manoeuvre.

2 Statistical Model Literature Study

The most recognised work on the statistics of the fluctuation of target radar cross section was conducted by Swirling [1]. The models proposed by Swerling considered two different forms of target, and also two different forms of radar processing/ target behaviour (temporal models).

The first of the target models assumed that the target RCS was created by the reflection from a group of distributed scatterers, each of a similar magnitude. The resulting RCS probability density function is an exponential distribution, which is also a χ^2 distribution with 2 degrees of freedom. The second target model assumes that within the group of scatterers, one of the scatterers has a larger magnitude and dominates the return. The RCS probability density function that results is a χ^2 distribution with 4 degrees of freedom.

The two alternative temporal models were to consider how the echoes from pulses fluctuate between observations, and therefore how well the pulse amplitudes correlate when integrated. The first of the temporal scenarios is where the target is small (i.e. not many wavelengths in size), or rotating only very slowly and therefore there is a negligible change in the RCS amplitude between pulses. The second scenario is where the target is very many wavelengths across and is rotating quickly, causing every pulse to have a different amplitude.

The reality is that modern targets, especially low-observable targets do not necessarily present statistical behaviours that are an exact match to either of the Swerling RCS distributions. Modern surveillance radars are also generally operating at higher carrier frequencies than when Swerling first performed his analysis, meaning that very small changes in target aspect are likely to cause some RCS decorrelation during the radar integration period.

Analysis of modern targets has revealed that the RCS profile can often be more accurately modelled using log-Normal distributions [2] or even Gaussian Mixture models [3], however the issues of how to provide representative control of correlation between pulses in a simple manner are not always addressed. Given that the correlation behaviour between pulses for low-observable targets may change dramatically with aspect [4], alternative methods that can recreate RCS characteristics are useful.

For design and analysis, extended Log-Normal, Weibull and Chi-Squared distributions have been employed to provide improved predictions of probability of detection when pulses are integrated by using behaviour models that use more degrees of freedom [5,6,7]. For use in simulation where more detailed models of the signal processing algorithms are being investigated however, rather than the raw detection statistics, what is often needed is an appropriate estimate of the target RCS value that changes appropriately on a pulse-to-pulse basis. The RCS value can then be combined with an appropriate radar path attenuation and clutter model, and a receiver noise and signal processing model, to create representative data samples for the radar algorithms. Therefore a model is needed that can generate correlated RCS samples very quickly and easily and that can capture the key statistical behaviours of the target as is both performs large manoeuvres between radar waveform dwells, and also on a pulse-to-pulse basis.

By definition, a simulation is not reality and therefore understanding how the choice of RCS model impacts on simulated results is important. As the Swerling models are well understood and widely accepted in industry, strategies where simulation runs are performed using Swerling models, and then repeated with alternative RCS statistical behaviours can be very instructive in identifying simulation characteristics that are very sensitive to the RCS model employed.

3 Weibull Approximations

The Weibull cumulative distribution function [8] is shown in Equation (1) as $F(x;\mu,k)$ for a sample from the distribution of x and for a distribution with a scale of μ and shape of k.

$$
\begin{aligned}
F(x;\mu,k) &= 1-\exp\left(-\left(\frac{x}{\mu}\right)^{k}\right) \\
x_{med} &= \mu\left(\ln(2)\right)^{\frac{1}{k}} \\
\therefore \mu &= \frac{x_{med}}{\ln(2)^{\frac{1}{k}}} \\
x_{med} &= x_{mean}\frac{\left(\ln(2)\right)^{\frac{1}{k}}}{\Gamma\left(1+\frac{1}{k}\right)}
\end{aligned}
\tag{1}
$$

For assessing the RCS distributions, the cumulative distribution is modified to allow probabilities of detection to be calculated directly and is shown as $P(X>x;\mu,k)$, which is the probability of observing a new random value X that is greater than the specified level of x as shown in Equation (2). A random value from the Weibull distribution to represent RCS is generated as W in Equation (2), where U is a uniform random variable in the interval [0,1].

$$
\begin{aligned}
P(X>x;\mu,k) &= \exp\left(-\left(\frac{x}{\mu}\right)^{k}\right) \\
W &= \mu\left(-\ln(U)\right)^{\frac{1}{k}}
\end{aligned}
\tag{2}
$$

The piecewise Weibull distribution is created by changing the scale and shape parameters μ and k for the regions above and below the median, providing two independent curve sections to approximate the RCS distribution. In generating a sample from a piecewise distribution, then the uniform random number U is generated in the interval [0,1] and tested to see if it is greater or less than 0.5, in order to determine which of the Weibull distribution section coefficients to employ (see Equation (3) to (7) for more details).

4 Statistical behaviour of RCS of targets

Two different aircraft targets and a ship target have been analysed using the POFacets [9] electromagnetic simulation software. POFacets uses a single-bounce physical optics approach to predicting the RCS of targets and although limited in its capabilities, produces RCS patterns that are representative of many targets, in particular low-observable vehicles that have been designed to minimise the multi-bounce reflections, which POFacets does not model. The POFacets data allows the piecewise Weibull method to be demonstrated, and is not intended to provide exact RCS results for specific targets.

For each target, 45000 sample points have been generated at random over a full 4π Steradian sphere (upper hemisphere for the ship target) and the RCS magnitudes analysed. The analysis has been conducted to investigate how the RCS varies in small local regions, and also how RCS varies over an entire view sphere. For the localised analysis, 300 conical sections have been generated each with a total cone angle of 5° (i.e. ±2.5° around the gyratrix of the cone). The RCS cumulative density function within each conical section has been analysed. For the localised analysis, the cumulative density has been normalised relative to the median of the data in the conical section, and then the spread of these 300 cumulative density functions analysed. For the bulk analysis, the 300 cone medians have been used to form a separate density estimate of how the RCS varies over the full sphere.

The analysis of the conventional aircraft shown in Figure 1 at 15GHz are shown in Figure 2 for the normalised local behaviours, and for the bulk statistic behaviours in Figure 3. To generate a representative target RCS, a sample is first drawn from the normalised local distribution in Figure 2, then multiplied by a sample drawn from the bulk distribution shown in Figure 3. The samples drawn from the local distribution in Figure 2 will vary on a pulse-by-pulse basis, whereas the samples from the bulk distribution in Figure 3 will usually only need to be reselected for each new integration interval or significant change of the target characteristics.

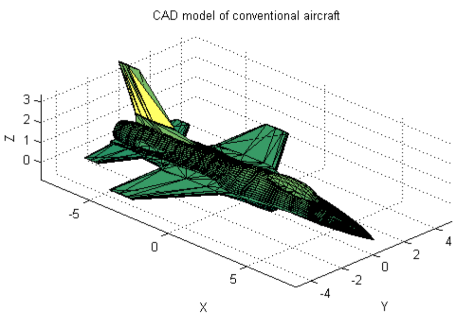

Figure 1: CAD model of conventional aircraft

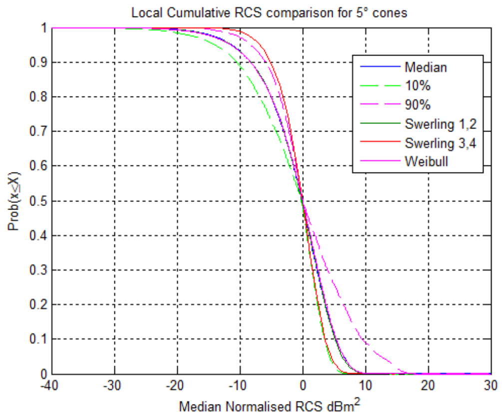

Figure 2: Normalised Local CDF comparison for conventional aircraft

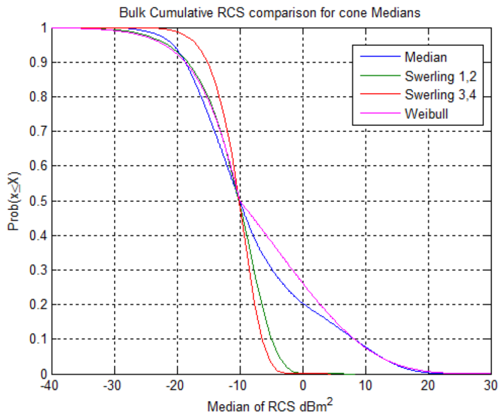

Figure 3: Bulk CDF of local medians for conventional aircraft

Therefore the model data can be used to mimic an RCS value by using the bulk RCS model to capture the 'slow' behaviour of the RCS changing with either view angle, change of carrier frequency, or due to RCS range-dependency at short ranges; the local behaviour can then be used to generate RCS samples that are representative of individual pulses within a waveform. The coefficients of the piecewise Weibull bulk and local regions for the conventional aircraft are:

Local Coefficient			Bulk Coefficient			
x_{med}	k_L	k_H	x_{med}	k_L	k_H	x_{min}
0dBm^2	0.99	0.97	-10.0dBm^2	0.94	0.28	-25dBm^2

Where k_L is the shape coefficient for RCS values less than the median, k_H is the shape coefficient to use for RCS values higher than the median, and x_{med} is the median value for the Weibull distribution to use to calculate μ_L and μ_H using Equation (1) (see equation (7) for more details of how k_L and k_H are employed and equation (8) to see how they are calculated).
It is interesting to note that for the local behaviours, the Swerling 1 and 2 model behaviour is a very good approximation to the median statistics across all 300 cone regions (k=1 would be a match to Swerling 1 and 2). For the bulk RCS however, for RCS values less than the median, the Swerling 1 model is again reasonable, but a very long-tailed distribution is needed to model larger RCS values above the median indicated by a *k* value much less than unity.

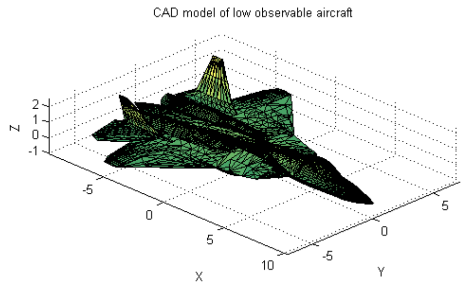

Figure 4: CAD model of low-observable aircraft

The analysis of the low-observable aircraft shown in Figure 4 at 15GHz are shown in Figure 5 for the normalised local behaviours, and for the bulk statistic behaviours in Figure 6. The coefficients of the bulk and local regions for the low-observable aircraft are:

Local Coefficient			Bulk Coefficient			
x_{med}	k_L	k_H	x_{med}	k_L	k_H	x_{min}
0dBm^2	1.0	0.95	-15.9dBm^2	0.58	0.18	-39dBm^2

The Weibull parameters for the local region of the low observable aircraft are very similar to the Swerling 1 statistics seen for the conventional aircraft, but for the bulk statistics, both the low and high regions deviate by a large margin from the Swerling model behaviours. Although the Weibull curve is not a 'perfect' match to the POFacets generated RCS statistics, the use of the piecewise Weibull function is still very much closer to the RCS bulk data than if the Swerling models are employed.
The Weibull models are primarily intended to be used to generate representative RCS values in a simulation, rather than to provide analytical analysis of probability of detection;

when probability of detection is concerned, the shape of the curve above the median is of little consequence for P_d<50%, and only has limited impact for P_d<70%. For pulse-to-pulse simulation for testing signal processing algorithms, the RCS fluctuation over the full range is of interest however. Equation (1) can be used to generate appropriate x_{med} median value for the bulk statistics from a desired mean RCS and the curves then offset if different target sizes are required to be simulated.

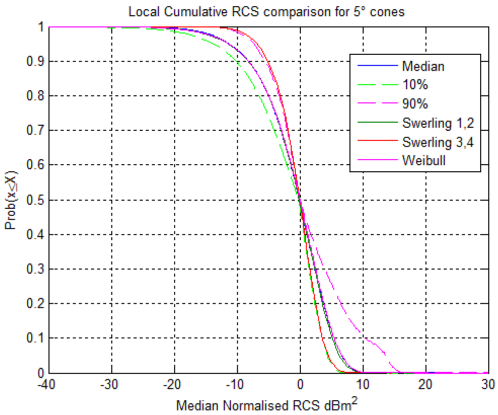

Figure 5: Normalised Local CDF comparison for low-observable aircraft

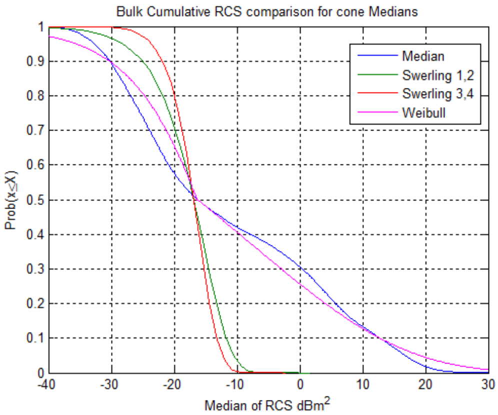

Figure 6: Bulk CDF of local medians for low-observable aircraft

The coefficients of the bulk and local regions for the ship target are:

Local Coefficient			**Bulk Coefficient**			
x_{med}	k_L	k_H	x_{med}	k_L	k_H	x_{min}
0dBm^2	0.87	0.59	21.4dBm^2	0.94	0.53	3dBm^2

The Weibull parameters for the local region of the ship are interesting in that the spread of RCS values below the median is similar to the Swerling 1 and 2 distribution, however above the median, there is more deviation indicating a more long-tailed distribution. For the bulk statistics, there is a better match to the Swerling 1 and 2 for values below the median, but again a long-tailed distribution is observed for values above the median.

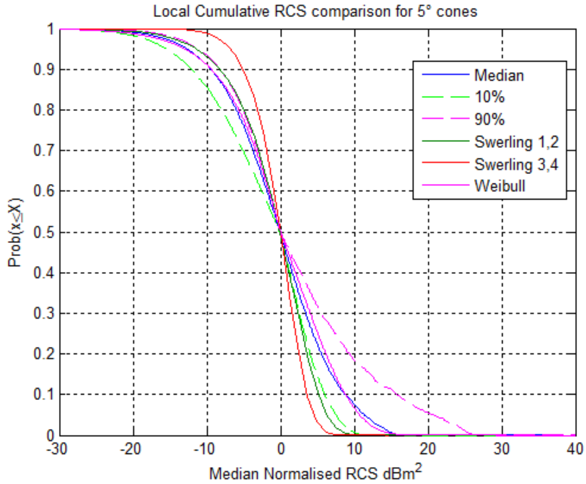

Figure 7: Normalised Local CDF comparison for ship

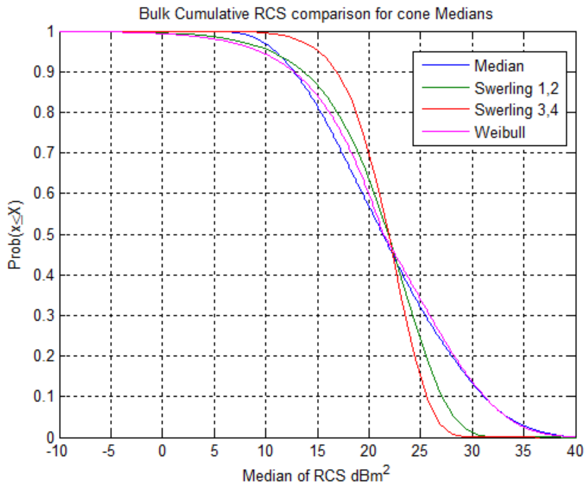

Figure 8: Bulk CDF of local medians for ship

6 Generating Samples from RCS Distribution

The Swerling modelling approach not only captured the distribution of the instantaneous RCS behaviour for a single pulse, but also attempted to provide models of the correlation between pulses when the radar performed integration. As the piecewise Weibull process is intended for use in simulation rather than analytical design analysis, it is appropriate to employ a more flexible correlation modelling mechanism.

The RCS statistical analysis revealed that for a localised RCS region, the median behaviour is often very close to the Swerling 1 and 2 behaviour and the RCS scintillation within an integration period would be drawn from this 'local' distribution with a correlation between samples that is related to the target electrical size and rotation rate. Between processing intervals however, the overall median radar cross section may vary in a very non-Swerling manner and may have a very different temporal correlation behaviour.

A mechanism is desired that allows RCS sample values to be generated from the 'local' distribution for each pulse with a desired correlation behaviour, and also from the 'bulk' RCS distribution with a different temporal correlation pattern.

A convenient method for generating temporally correlated samples from any distribution is through the use of a Memoryless Non-Linear Transform (MNLT) process [10]. The MNLT method has been used for modelling correlated clutter samples, but is also very applicable to the modelling of RCS sample correlation. The MNLT process operates by generating random samples from a Gaussian distribution, and then using a filtering process to impart a desired spectral density, which will therefore impart a temporal correlation relationship. Often for radar modelling, the spectral shaping will be in the form of a low-pass filtering operation of the Gaussian samples. As low-pass filtering is a convolution process (i.e. multiply samples by a window, and then sum), the central limit theorem causes the Gaussian samples after filtering to remain Gaussian distributed. As the probability density of the samples is known to still be Gaussian, then a mapping function can be applied that transforms the Gaussian samples into any other desired probability density function.

The choice of the Weibull distribution for mimicking the statistical behaviour of the RCS was influenced by the simplicity of implementing a transform from filtered Gaussian samples into the desired Weibull statistics.

The generation of a Weibull sample employs two streams of independent random numbers generated from a Gaussian Normal distribution noted as $N_1(t)$ and $N_2(t)$, where t is the time sample index in practice. Both streams are passed independently through the filter function represented by the impulse response I(t). The filter could be a Finite impulse response filter, where I(t) would represent the filter coefficient vector directly, or an Infinite impulse response filter process where the impulse response can be evaluated numerically. Equation (3) details the filtering process to obtain one of the Gaussian data streams that maintains the unity variance properties of a Normal distribution, but gains inter-sample correlation properties, where '*' is the process of convolution.

$$G(t) = \frac{I(t) * N(t)}{\sqrt{\int_0^\infty I(t)^2 dt}} \qquad (3)$$

If the latest values of the two filtered Gaussian streams $G_1(t)$ and $G_2(t)$ are squared and summed, the resulting sample will follow a Chi-Squared distribution with two degrees of freedom. For the generation of the Weibull random samples, we ideally need a sample drawn from a uniform distribution; luckily the conversion from the Chi-Squared sample to a uniform sample may be achieved quite simply using the non-linear transform detailed in the derivation in Equation (4) and (5), where $F(y,v)$ describes the cumulative distribution function for a Chi-Squared distributed variable y with v degrees of freedom [11], $\Gamma(.)$ is the Gamma function, and $\gamma(s,q)$ is the lower incomplete Gamma function.

$$\begin{aligned} &F(y,v) = \frac{1}{\Gamma\left(\frac{v}{2}\right)}\gamma\left(\tfrac{v}{2},\tfrac{y}{2}\right) \\ &\text{where} \\ &\gamma(s,q) = \int_0^q t^{s-1}e^{-t}dt \end{aligned} \qquad (4)$$

For v=2 degrees of freedom,

$$\begin{aligned} &F(y,2) = \frac{1}{\Gamma(1)}\gamma\left(1,\tfrac{y}{2}\right) \\ &\text{where } \Gamma(1) = 1 \text{ and} \\ &\gamma(1,q) = \int_0^q e^{-t}dt = 1 - e^{-q} \\ &\therefore F(y,2) = 1 - e^{-\frac{y}{2}} \end{aligned} \qquad (5)$$

From Equation (5), we can now see that if we generated a random number by squaring and summing two Normally-distributed random values, then the cumulative distribution $F(y,2)$ describes the probability of the generated random number being less than a reference value y. If we replaced the reference y in Equation (5) with the generated random number, then the transformed number that results would now still be random, but follow a uniform distribution. As the output distribution is uniform (denote by the distribution "U"), then $p=1-U$ is still uniformly distributed as indicated in Equation (6):

$$\begin{aligned} &F(y,2) = 1 - e^{-\frac{y}{2}} = U \Rightarrow \text{Uniformly Distributed} \\ &1 - U = e^{-\frac{y}{2}} = p \Rightarrow \text{Uniformly Distributed} \end{aligned} \qquad (6)$$

Therefore the Memoryless Non-Linear Transform proceeds by mixing the two filtered Gaussian data stream into a Chi-Squared distribution by combining the magnitude of the two Gaussian samples (i.e. the parameter z in (7) will be Chi-Squared distributed), and then to a uniform distribution as the parameter p. The uniformly distributed data in p are mapped using the cumulative distribution of the Weibull distribution into the RCS value representation W. To achieve the piecewise split of the model, the value p is tested to determine which half of the model curve it lies in, and the appropriate μ and k coefficients selected to represent the high value or low value RCS model sections. Two separate MNLT processes, one for the localised RCS response and one for the global response will be required, resulting in two filter impulse functions and a total of four streams of Normal distributed numbers being required, resulting in two Weibull values, one changing on a pulse-to-pulse basis, W_{local}, and one changing with each waveform, W_{global}. In practice, a sample drawn from an analytic Weibull distribution will have a very small probability of being a very small value approaching zero. In reality, real targets will not have their RCS fade completely and therefore a small correction factor, x_{min}, has been added to the generated RCS values in order to provide a practical lower limit to the modelled RCS. The full process is described in Equation (7), based on the results from Equations (2), (3) and

(6) with the RCS samples in W having units of metres squared.

$$\begin{aligned}
&z = \frac{G_1^2 + G_2^2}{2} \\
&p = \exp(-z) \quad \Rightarrow \text{Uniformly Distributed} \\
&W = \mu(-\ln(p))^{\frac{1}{k}} \quad \text{For Weibull} \\
&\therefore \text{When splitting into sections :} \\
&W = \begin{cases} \mu_H z^{\frac{1}{k_H}} & p \le 0.5 \\ \mu_L z^{\frac{1}{k_L}} & p > 0.5 \end{cases} \\
&RCS = W_{local} \cdot W_{Global} + x_{\min}
\end{aligned} \tag{7}$$

To generate the x_{med}, μ and k values from the raw RCS samples, the 10th, 50th and 90th percentile RCS values are captured as x_{10}, x_{med} and x_{90} and by rearranging (1) and solving the simultaneous equations, Equation (8) results.

$$\begin{aligned}
&b = \ln(-\ln(0.5)) \\
&\alpha_L = \ln(-\ln(0.1)), \ \ \alpha_H = \ln(-\ln(0.9)) \\
&\mu_L = \frac{\exp(\alpha_L \ln(x_{med}) - b\ln(x_{10}))}{\alpha_L - b} \\
&k_L = \frac{b}{\ln(x_{med}) - \ln(\mu_L)} \\
&\mu_H = \frac{\exp(\alpha_H \ln(x_{med}) - b\ln(x_{90}))}{\alpha_H - b} \\
&k_H = \frac{b}{\ln(x_{med}) - \ln(\mu_H)}
\end{aligned} \tag{8}$$

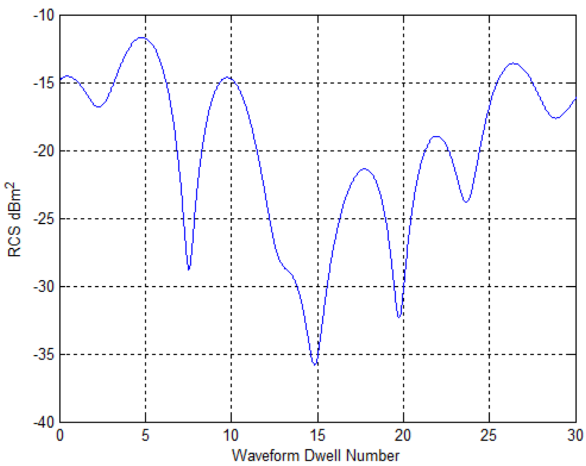

Figure 9: RCS simulation for low-observable aircraft.

A simulation has been conducted for the low-observable aircraft where the aircraft is observed with a LPRF of 7500Hz that is integrated for 10ms for each waveform dwell. The local RCS variation is modelled as fluctuating with a bandwidth of 13.3Hz, and the bulk median RCS fluctuating with a bandwidth of 0.8Hz in order to simulate the typical RCS variation with changing range that may be experienced by a missile seeker at a few kilometres from the target. The variation in RCS over 30 waveform dwells is shown in Figure 9. It is clear from the figure that for many of the dwells, the RCS scintillation during the waveform period is small, however when RCS fades occur such as during waveform number 16, variations of 10dB or so may be observed, which can impact on the radar signal processing stages.

7 Conclusions

The piecewise Weibull approach to modelling the RCS statistics is simple and allows non-Swerling behaviours to be approximated, as well as capturing the behaviour of targets that can be approximated by the classic Swerling statistical behaviours. The use of the Weibull distribution in combination with a memoryless non-linear transform approach can produce controlled temporal correlation between the RCS scintillations that are modelled.

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